CHAPTER 1.4: SCALAR AND VECTOR

SCALAR AND VECTOR

Scalars

Scalars are quantities which have magnitude without direction

Examples of scalars

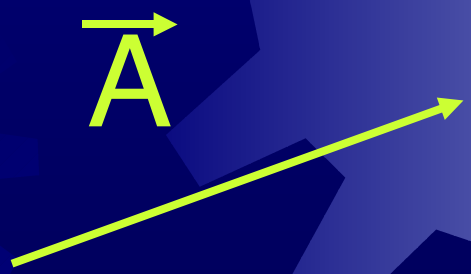
- temperature
- mass
- kinetic energy
- time
- amount
- density
- charge

Vector

A vector is a quantity that has both magnitude (size) and direction

it is represented by an arrow whereby
- the length of the arrow is the magnitude, and
- the arrow itself indicates the direction

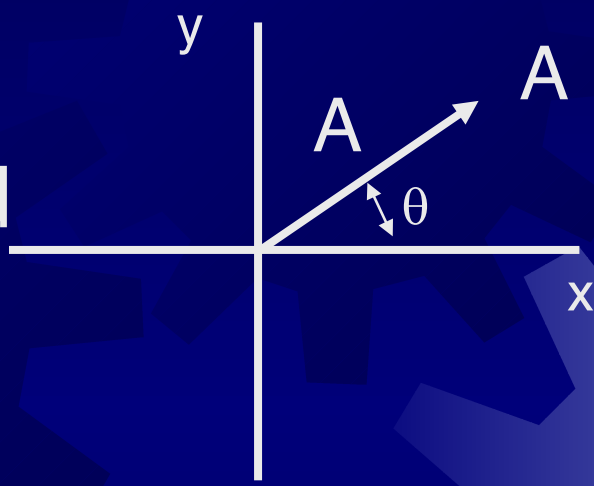
The symbol for a vector is a letter with an arrow over it



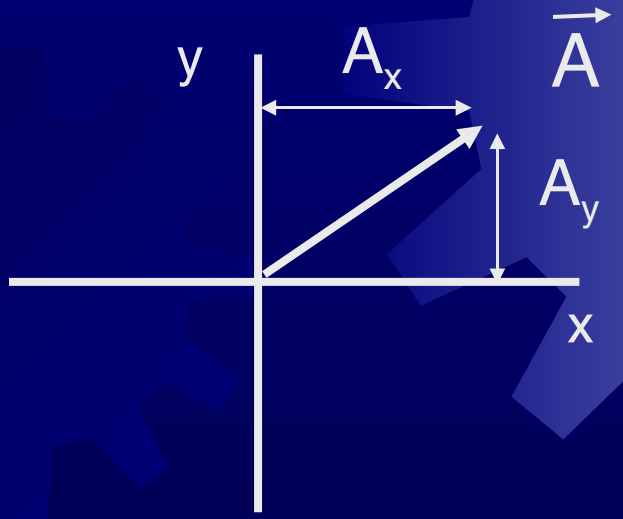
Example

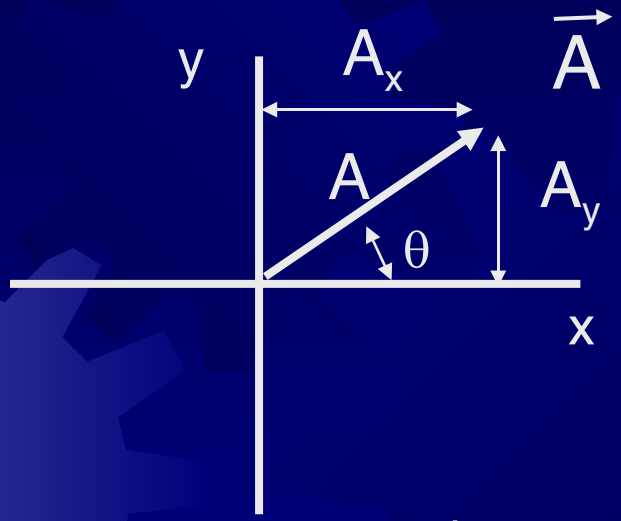
Two ways to specify a vector

It is either given by
- a magnitude A, and
- a direction θ



Or it is given in the x and y components as
- A_x
- A_y





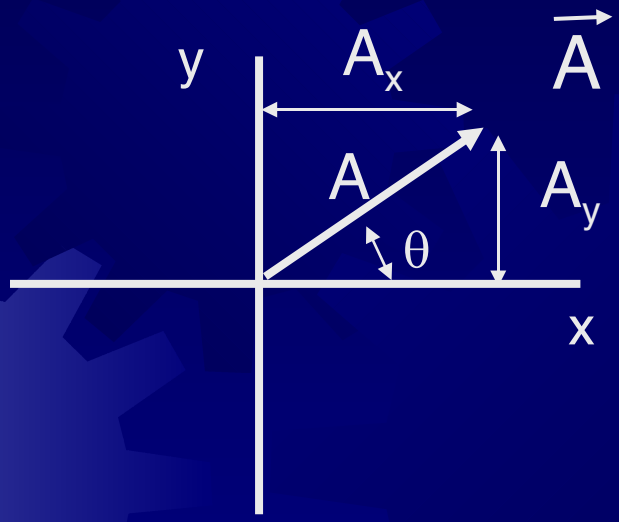
$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

The magnitude (length) of A is found by using the Pythagorean Theorem

$$|A| = \sqrt{(A_x^2 + A_y^2)}$$

The length of a vector clearly does not depend on its direction.



The direction of $\vec{A}$ can be stated as

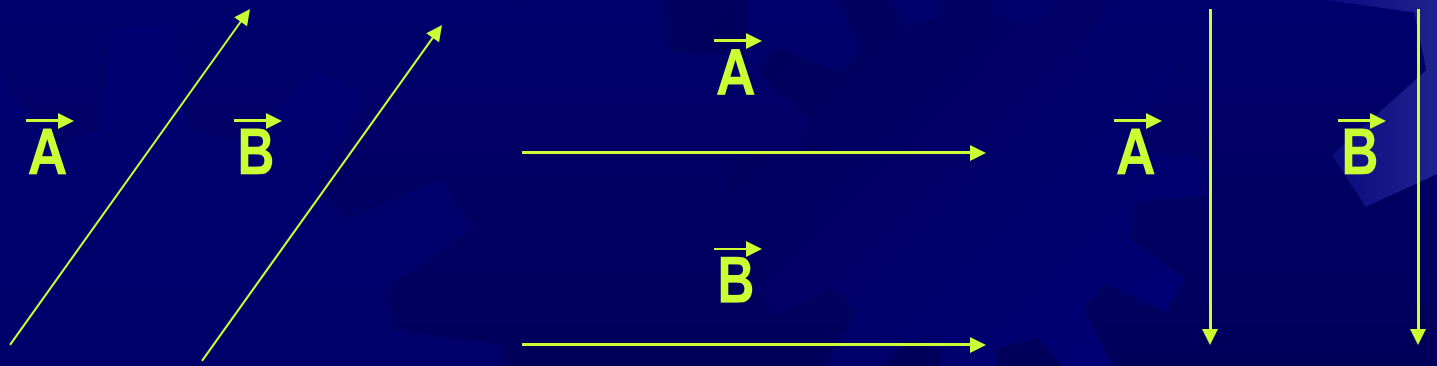
$$\tan \theta = Ay / Ax$$

$$\theta = \tan^{-1}(Ay / Ax)$$

Some Properties of Vectors

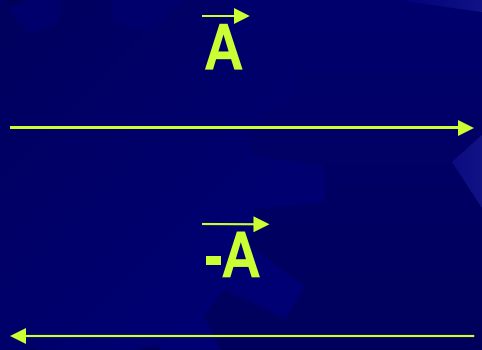
Equality of Two Vectors

Two vectors $\vec{A}$ and $\vec{B}$ may be defined to be equal if they have the same magnitude and point in the same directions. i.e. $\vec{A} = \vec{B}$



Negative of a Vector

The negative of vector $\vec{A}$ is defined as giving the vector sum of zero value when added to $\vec{A}$. That is, $\vec{A} + (-\vec{A}) = 0$. The vector $\vec{A}$ and $-\vec{A}$ have the same magnitude but are in opposite directions.



Scalar Multiplication

The multiplication of a vector A by a scalar α

- will result in a vector B

$$B = \alpha A$$

- whereby the magnitude is changed but not the direction

- Do flip the direction if α is negative

$$B = \alpha A$$

If $\alpha = 0$, therefore $B = \alpha A = 0$, which is also known as a zero vector

$$\beta(\alpha A) = \alpha\beta A = \alpha(\beta A)$$

$$(\beta+\alpha)A = \alpha A + \beta A$$

Example

Vector Addition

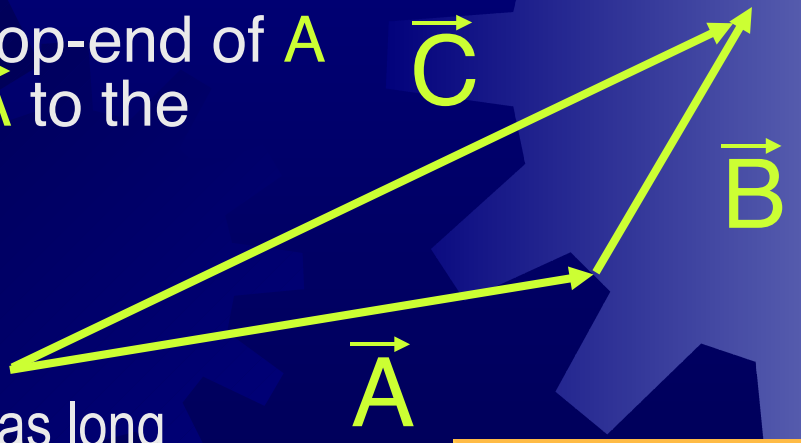
The addition of two vectors $\vec{A}$ and $\vec{B}$

- will result in a third vector $\vec{C}$ called the resultant

$$\vec{C} = \vec{A} + \vec{B}$$

Geometrically (triangle method of addition)

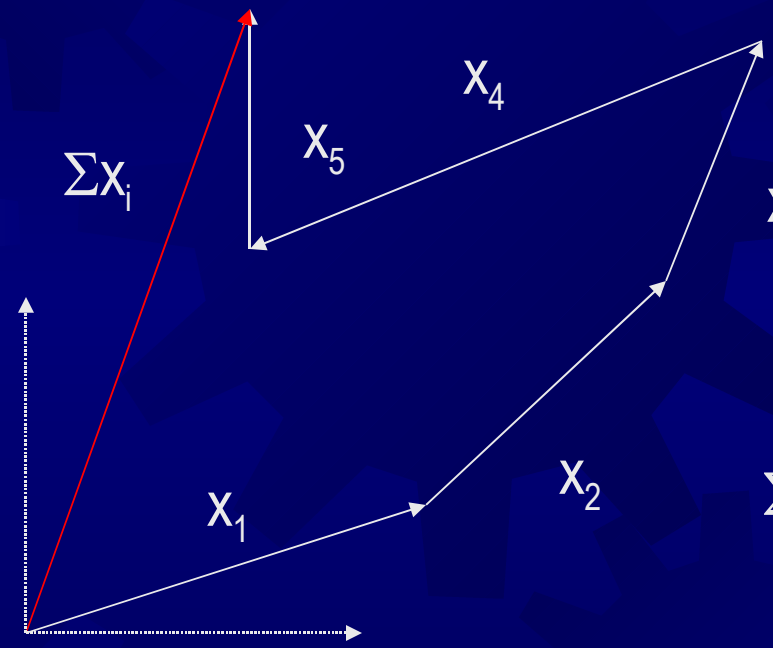
- put the tail-end of $\vec{B}$ at the top-end of $\vec{A}$
- $\vec{C}$ connects the tail-end of $\vec{A}$ to the top-end of $\vec{B}$



We can arrange the vectors as we like, as long as we maintain their length and direction

Example

More than two vectors?



$$\Sigma x_i = x_1 + x_2 + x_3 + x_4 + x_5$$

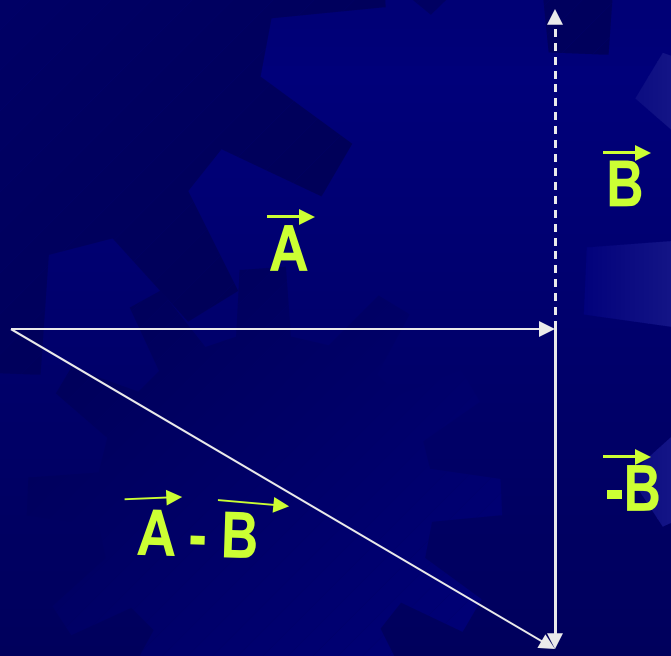
Example

Vector Subtraction

Equivalent to adding the negative vector

$$\vec{C} = \vec{A} - \vec{B}$$

$$= \vec{A} + (-\vec{B})$$

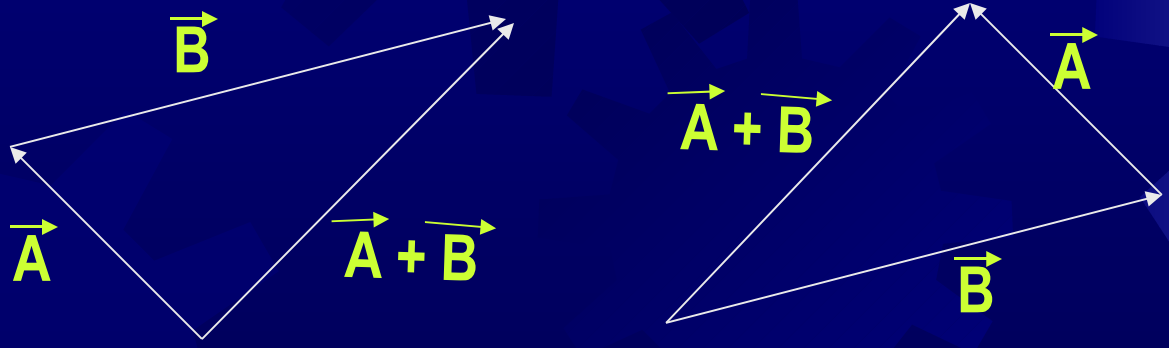


Example

Rules of Vector Addition

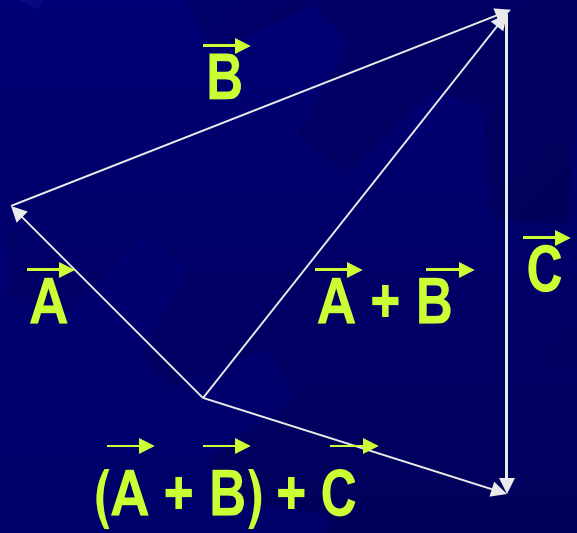
- commutative

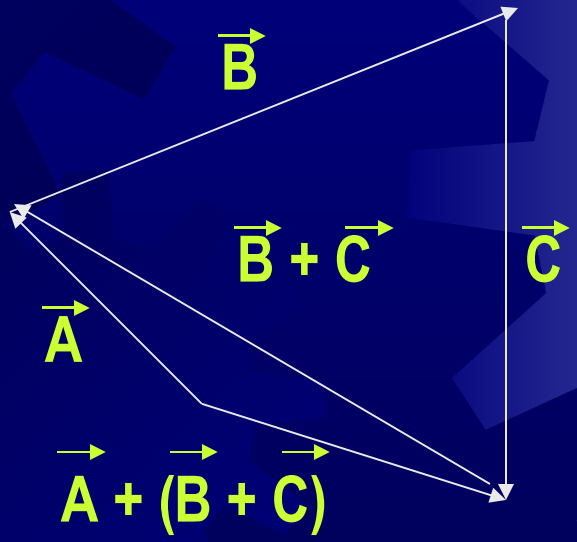
$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$



- associative

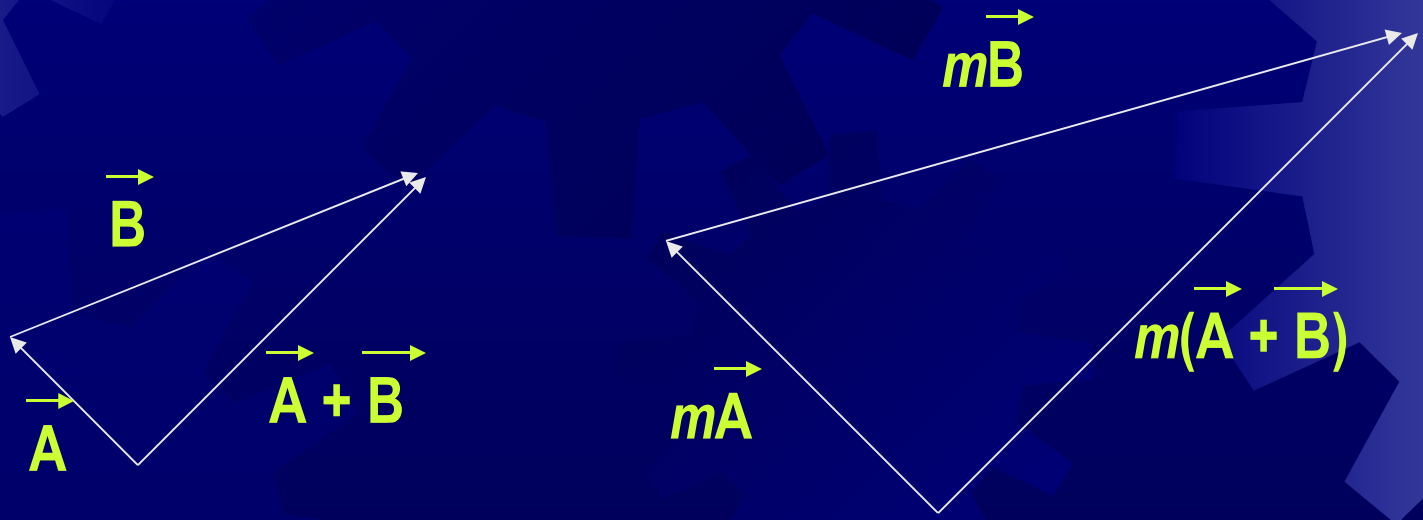
$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$



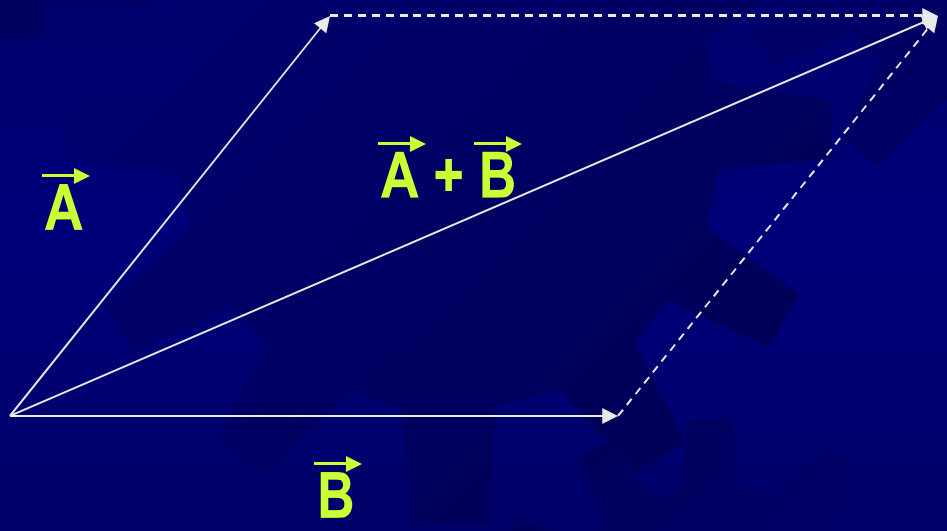


- distributive

$$m(\vec{A} + \vec{B}) = m\vec{A} + m\vec{B}$$



Parallelogram method of addition (tailtotail)



The magnitude of the resultant depends on the relative directions of the vectors

Unit Vectors

- a vector whose magnitude is 1 and dimensionless
- the magnitude of each unit vector equals a unity; that is, $|\hat{i}|$ $|\hat{j}|$ $|\hat{k}|$ = 1

and defined as

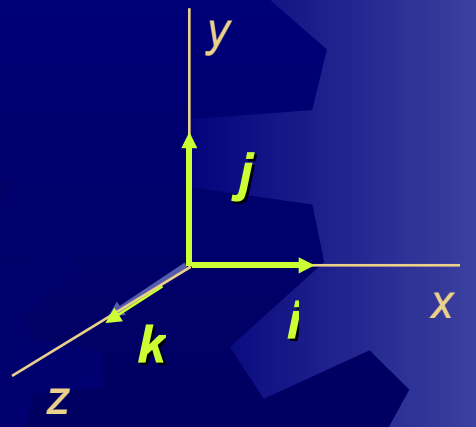
$\hat{i}$ a unit vector pointing in the x direction

$\hat{j}$ a unit vector pointing in the y direction

$\hat{k}$ a unit vector pointing in the z direction

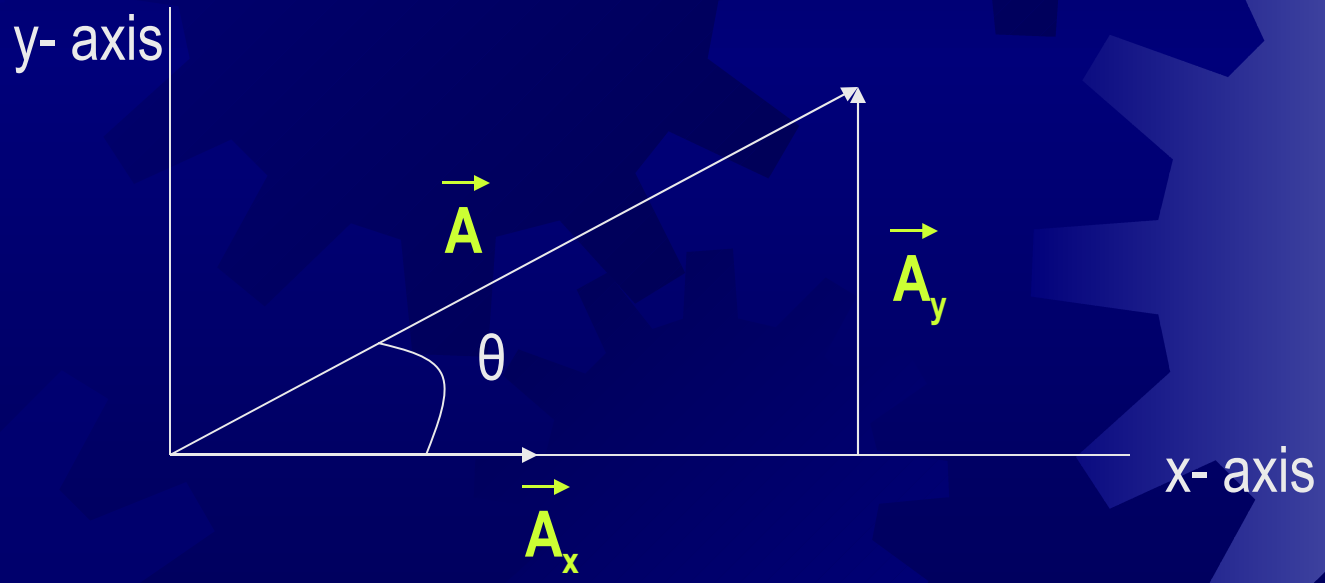
Useful examples for the Cartesian unit vectors [***i, j, k***]

- they point in the direction of the *x*, *y* and *z* axes respectively



Component of a Vector in 2-D

- vector $\vec{A}$ can be resolved into two components $\vec{A}_x$ and $\vec{A}_y$

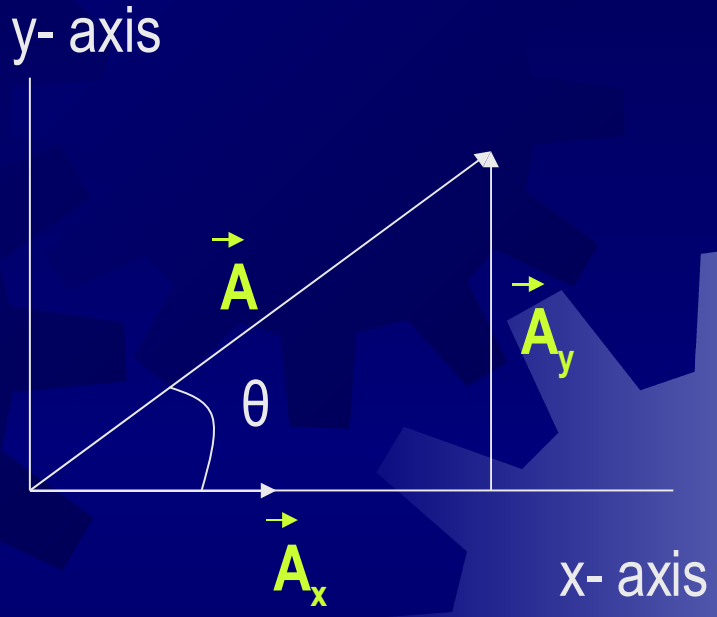


$$\vec{A} = \vec{A}_x + \vec{A}_y$$

The component of A are

$$|\vec{A}_x| = A_x = A\cos\theta$$

$$|\vec{A}_y| = A_y = A\sin\theta$$



The magnitude of A

$$A = \sqrt{A_x^2 + A_y^2}$$

The direction of A

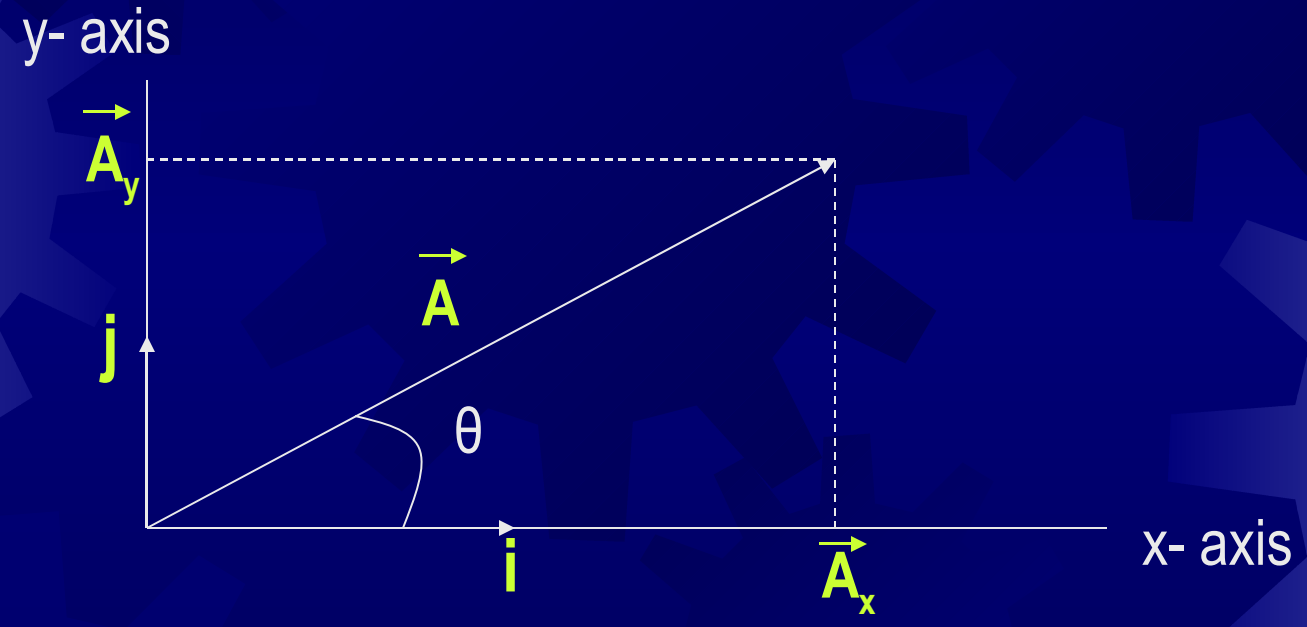
$$\tan\theta = Ay / Ax$$

$$\theta = \tan^{-1}(Ay / Ax)$$

Example

The unit vector notation for the vector $\vec{A}$ is written

$$\vec{A} = A_x i + A_y j$$

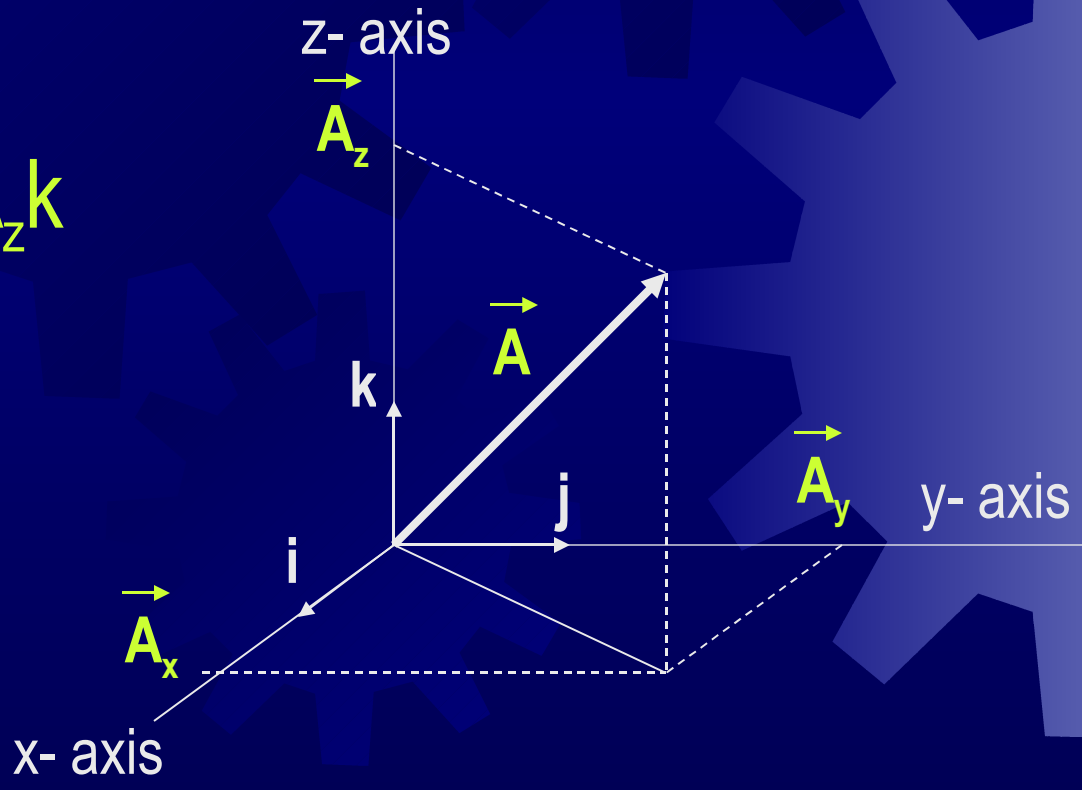


Example

Component of a Vector in 3-D

- vector $\vec{A}$ can be resolved into three components $\vec{A}_x$, $\vec{A}_y$ and $\vec{A}_z$

$$\vec{A} = A_x i + A_y j + A_z k$$



if

$$\vec{A} = A_x i + A_y j + A_z k$$

$$\vec{B} = B_x i + B_y j + B_z k$$

$\vec{A} + \vec{B} = \vec{C}$ sum of the vectors A and B can then be obtained as vector $\vec{C}$

$$\vec{C} = (A_x i + A_y j + A_z k) + (B_x i + B_y j + B_z k)$$

$$= (A_x + B_x)i + (A_y + B_y)j + (A_z + B_z)k$$

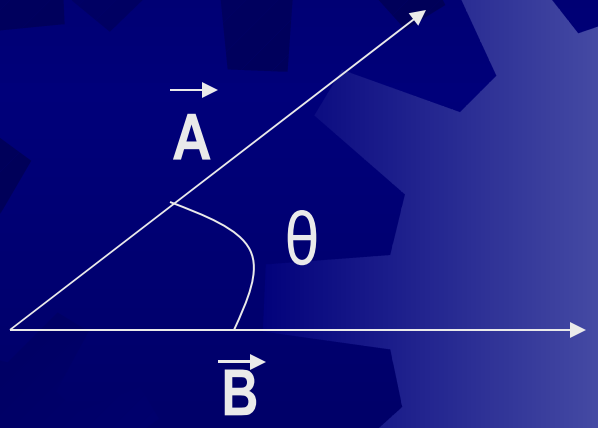
$$= C_x i + C_y j + C_z k$$

Example

Dot product (scalar) of two vectors

The definition:

$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta$$



Dot product (scalar product) properties:

- **if θ = 90⁰ (normal vectors) then the dot product is zero**

$$|\vec{A} \cdot \vec{B}| = AB \cos 90 = 0 \quad \text{and} \quad i \cdot j = j \cdot k = i \cdot k = 0$$

- **if θ = 0⁰ (parallel vectors) it gets its maximum value of 1**

$$|\vec{A} \cdot \vec{B}| = AB \cos 0 = 1 \quad \text{and} \quad i \cdot j = j \cdot k = i \cdot k = 1$$

- the dot product is commutative

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

- Use the distributive law to evaluate the dot product if the components are known

$$\vec{A} \cdot \vec{B} = (A_x i + A_y j + A_z k) \cdot (B_x i + B_y j + B_z k)$$

$$= (A_x B_x)\, i.i + (A_y B_y)\, j.j + (A_z B_z)\, k.k$$

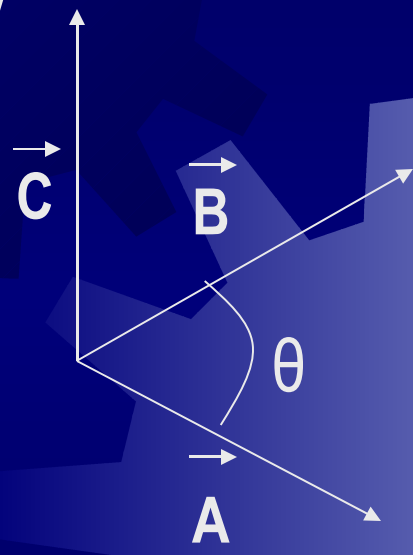
$$= A_x B_x + A_y B_y + A_z B_z$$

Example

Cross product (vector) of two vectors

The magnitude of the cross product given by

$$|\vec{C}| = |\vec{A} \times \vec{B}| = |\vec{A}|\,|\vec{B}| \sin\theta$$



- the vector product creates a new vector
- this vector is normal to the plane defined by the original vectors and its direction is found by using the right hand rule

Cross product (vector product) properties:

- **if θ = 0^0 (parallel vectors) then the cross product is zero**

$$|\vec{A} \times \vec{B}| = AB \sin 0 = 0 \quad \text{and} \quad i \times i = j \times j = k \times k = 0$$

- **if θ = 90^0 (normal vectors) it gets its maximum value**

$$|\vec{A} \times \vec{B}| = AB \sin 90 = 1 \quad \text{and} \quad i \times i = j \times j = k \times k = 1$$

- the relationship between vectors i , j and k can be described as

$$i \times j = -j \times i = k$$

$$j \times k = -k \times j = i$$

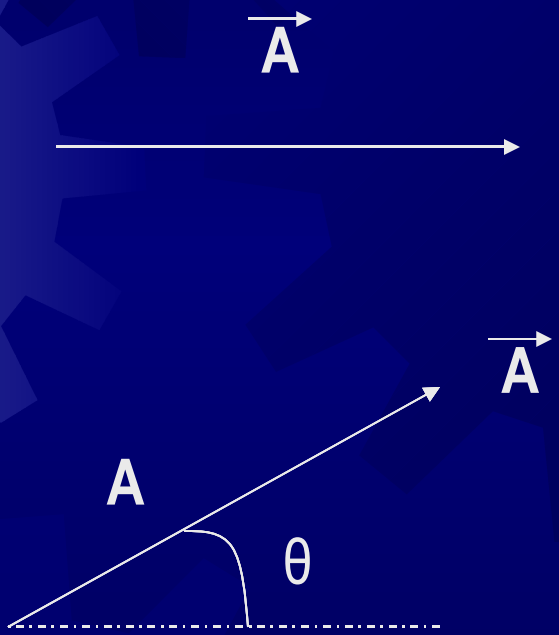
$$k \times i = -i \times k = j$$

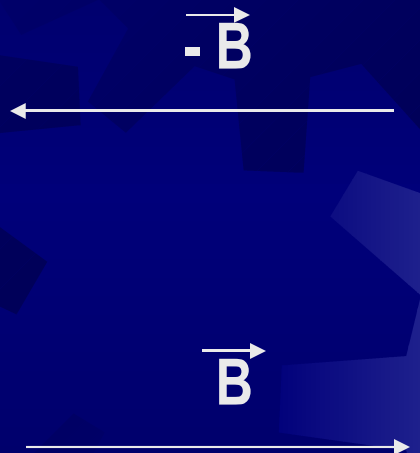
Example

Measurement and Error

THE END

Vectors are represented by an arrow





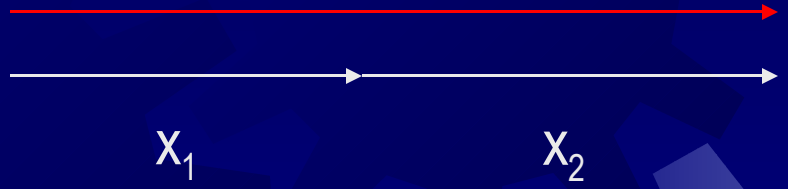
Conceptual Example

If B is added to A, under what condition does the resultant vector A + B have the magnitude equal to A + B ?

Under what conditions is the resultant vector equal to zero?

Example (1Dimension)

$x_1 + x_2$



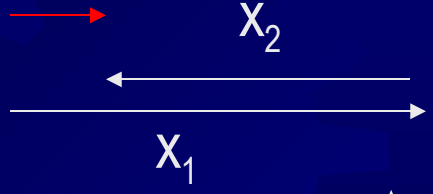
x_1 x_2

$x_1 + x_2 = 8$

$x_1 = 5$

$x_2 = 3$

$\Delta x = x_2 - x_1$



x_2

x_1

$\Delta x = x_2 - x_1 = 2$

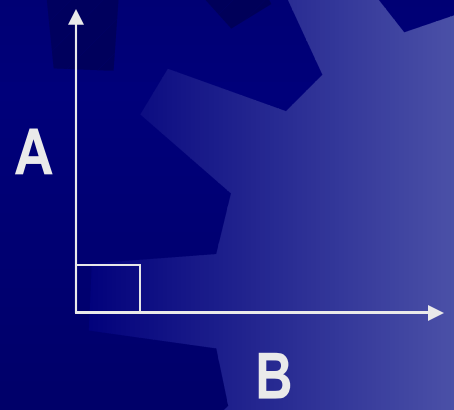
MORE EXAMPLE

Example 1 (2 Dimension)

If the magnitude of vector A and B are equal to 2 cm and 3 cm respectively , determine the magnitude and direction of the resultant vector, C for

a) A + B

b) 2A + B



SOLUTION

Solution

a) $|A + B| = \sqrt{A^2 + B^2}$

$= \sqrt{2^2 + 3^2}$

$= 3.6$ cm

The vector direction

$\tan \theta = B / A$

$\theta = 56.3$

a) $|2A + B| = \sqrt{(2A)^2 + B^2}$

$= \sqrt{4^2 + 3^2}$

$= 5.0$ cm

The vector direction

$\tan \theta = B / 2A$

$\theta = 36.9$

MORE EXAMPLE

Example 2 (A Vacation Trip)

A car travels 20.0 km due north and then 35.0 km in a direction 60^0 west of north. Find the magnitude and direction of the car's resultant displacement.

SOLUTION

Solution

The magnitude of R can be obtained using the law of cosines as in figure

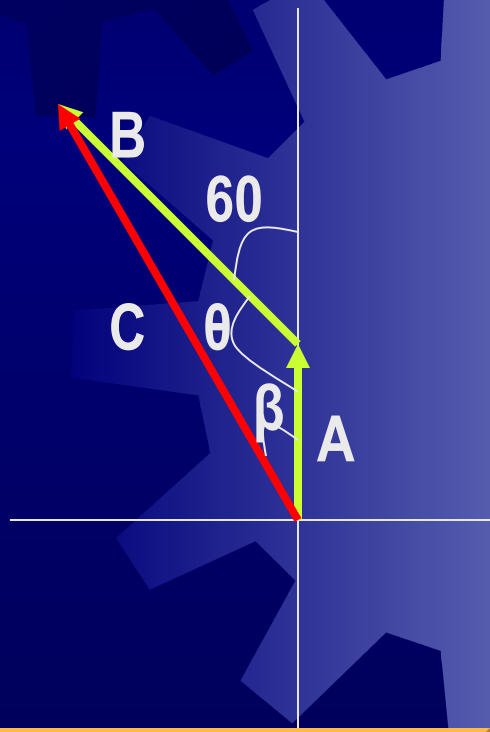
Since $\theta = 180^0 - 60^0 = 120^0$ and

$C^2 = A^2 + B^2 - 2AB \cos\theta$, we find that

$$C = \sqrt{A^2 + B^2 - 2AB \cos\theta}$$

$$C = \sqrt{20^2 + 35^2 - 2(20)(35) \cos 120^0}$$

$$C = 48.2 \text{ km}$$



Continue

The direction of C measured from the northerly direction can be obtained from the sines law

$$\frac{\sin\beta}{B} = \frac{\sin\theta}{C}$$

$$\sin\beta = \frac{B}{C}\sin\theta = \frac{35.0}{48.2}\sin 120^0 = 0.629$$

$$\beta = 38.9^0$$

Therefore, the resultant displacement of the car is 48.2 km in direction 38.9^0 west of north

Conceptual Example

*

If one component of a vector is not zero, can its magnitude be zero? Explain.

MORE EXAMPLE

Conceptual Example

If A + B = 0, what can you say about the components of the two vectors?

Example 1

Find the sum of two vectors A and B lying in the xy plane and given by

$$A = 2.0i + 2.0j \text{ and } B = 2.0i - 4.0j$$

SOLUTION

Solution

Comparing the above expression for A with the general relation $A = A_x i + A_y j$, we see that $A_x = 2.0$ and $A_y = 2.0$. Likewise, $B_x = 2.0$, and $B_y = -4.0$ Therefore, the resultant vector C is obtained by using Equation

$$C = A + B + (2.0 + 2.0)i + (2.0 - 4.0)j = 4.0i - 2.0j$$

or $C_x = 4.0$ $C_y = -2.0$

The magnitude of C given by equation

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{20} = 4.5$$

Exercise

Find the angle θ that C makes with the positive x axis

Example

A particle undergoes three consecutive displacements $d_1 = (1.5i + 3.0j – 1.2k)$ cm,

$d_2 = (2.3i – 1.4j – 3.6k)$ cm $d_3 = (-1.3i + 1.5j)$ cm. Find the component and its magnitude.

Solution

$R = d_1 + d_2 + d_3$

$= (1.5 + 2.3 – 1.3)i + (3.0 – 1.4 + 1.5)j + (-1.2 – 3.6 + 0)k$

$= (2.5i + 3.1j – 4.8k)$ cm

That is, the resultant displacement has component

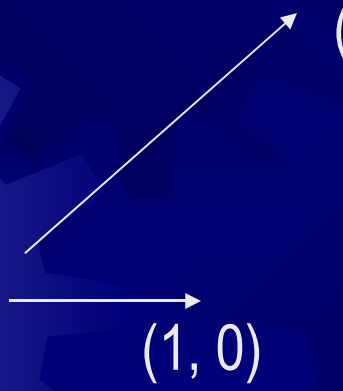
$R_x = 2.5$ cm $R_y = 3.1$ cm and $R_z = -4.8$ cm

Its magnitude is

$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$

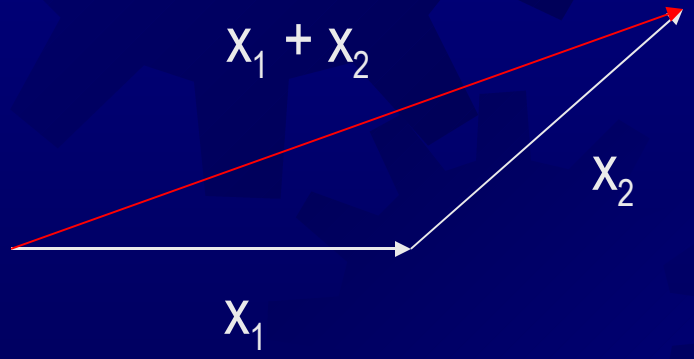
$= 6.2$ cm

Example - 2D [headtotail]

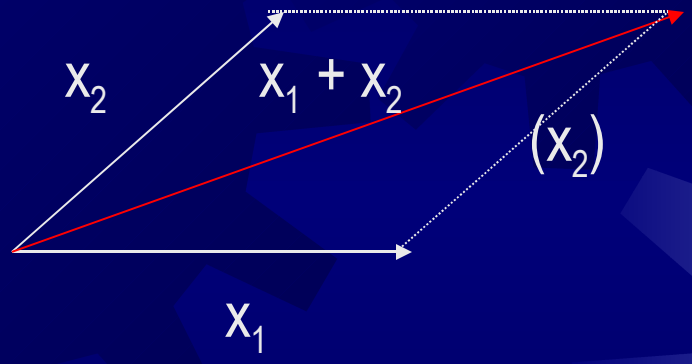


$x_1 + x_2 = (1, 0) + (2, 2)$

$= (3, 2)$



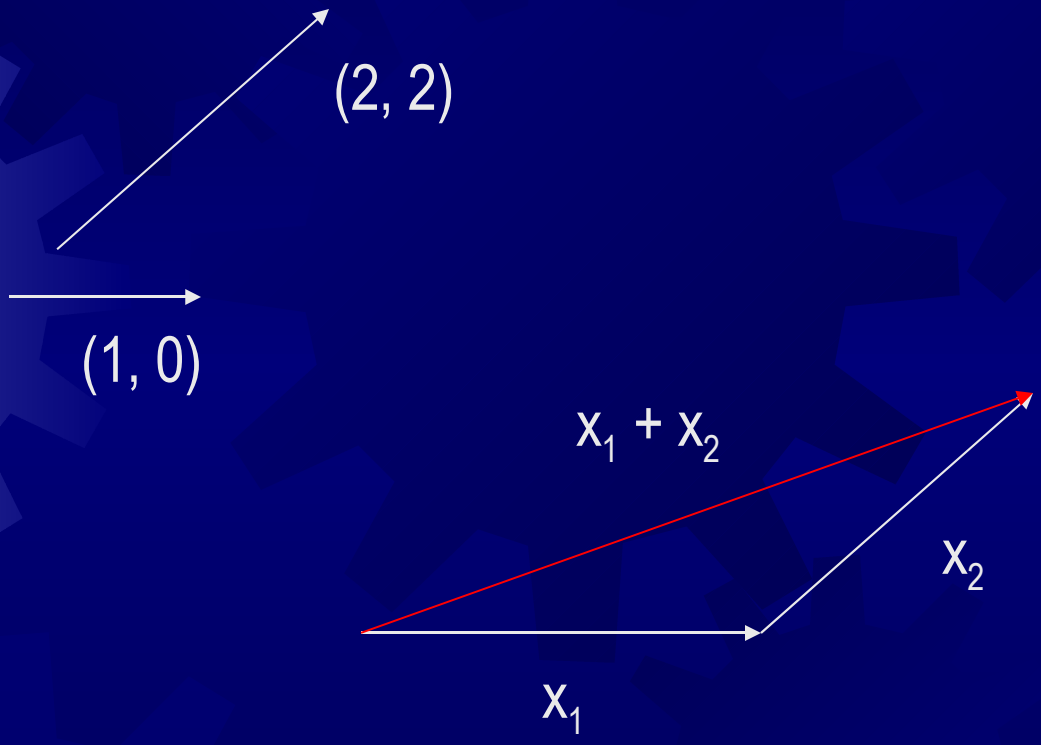
Example - 2D [tailtotail]



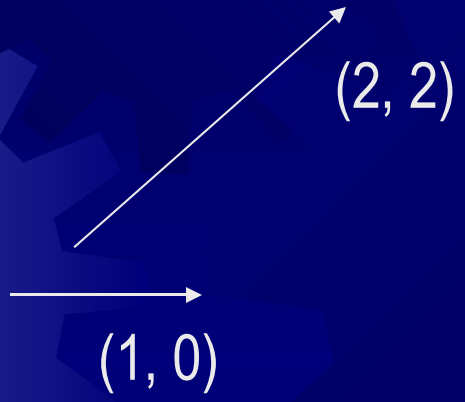
$x_1 + x_2 = (1, 0) + (2, 2)$
$= (3, 2)$

$x_1 - x_2$?

Example of 2D (subtraction)

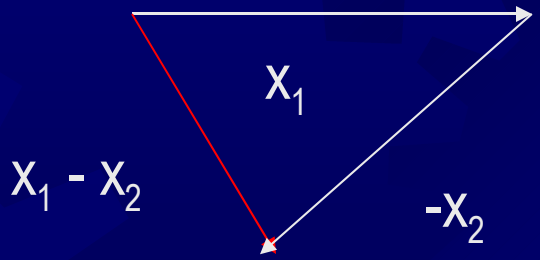


Example -2D for subtraction

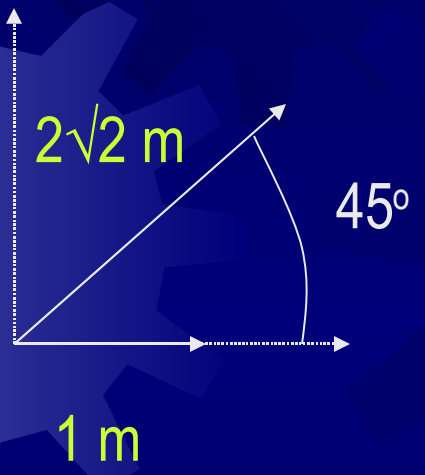


$x_1 - x_2 = x_1 + (-x_2)$

$x_1 - x_2 = (1, 0) - (2, 2)$
$= (-1, -2)$



Not given the components?

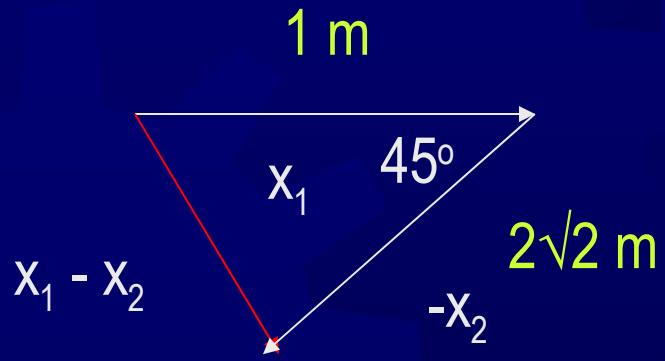


$X_1 = (1, 0)$

$X_2 = (x_{2E}, x_{2N})$

$= (2\sqrt{2}\cos(45^o), 2\sqrt{2}\sin(45^o))$

$= (2, 2)$



Cosine rule:

$a^2=b^2 + c^2 - 2bc\cos A$

$= 1 + 8 - 2\sqrt{2}(1/\sqrt{2})$

$a = \sqrt{5}$ m